Neutron Stars

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Several authors[1-5] have suggested that the recently discovered[5-7] extra-terrestrial sources of x-rays may be hot neutron stars. The plausibility of this suggestion, and in fact the likelihood that astronomers will ever be able to observe neutron stars by their x-ray emission, depend critically upon the cooling times of the hot stars. The main purposes of this note are: (i) to present the results, and suggest the implications, of some approximate calculations for the neutrino cooling rates of neutron stars; and (ii) to point out that some current ideas regarding the constituents of neutron stars should be revised. Our description of the states of a neutron star and the reactions by which it cools differ from the work of previous authors[2,3,8] in that we include in an approximate, self-consistent way the effects of the strong interactions among all the hadrons (strongly interacting particles) present. The principal new results obtained (for densities not much greater than nuclear densities) are[9]: (i) the existence of effective masses for all the hadrons; (ii) differential shifts in the threshold densities at which various kinds of particles are produced; and (iii) much faster cooling rates than previous workers have estimated. At densities greater than ten times nuclear densities,

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unsolved matters of principle are of primary importance.[10] We have therefore attempted to phrase our initial questions in terms of quantities that can be defined independent of any specific model for the interactions among the particles that constitute a neutron star. Our practical results are, of course, calculated on the basis of a specific model that has a limited domain of validity which we attempt to estimate.

The ground state of a neutron star can be determined[8] by minimizing the total energy subject to the constraints of conservation of charge and baryon number. Other authors[2,3,8] have adopted a noninteracting gas model for all the particles in the star in order to carry out this minimization; their approach is valid only in the low density limit. We have included the effects of the strong interactions in an independent pair model, I.PA.M., which is similar to the self-consistent independent pair model used by Gomes et al.[11] for discussing the ground state of nuclear matter. We suggest that a necessary criterion for the validity of any independent particle model is that the average interparticle separation, d, between hadrons satisfy the following inequality:

$$d > 0.5 \times 10^{-13} \text{ cm.} \tag{1}$$

If inequality (1) is not satisfied (i.e., the stellar density is $\gtrsim$ eight times nuclear densities), then pairs of hadrons spend most of their time within each other's hard cores and the concept of distinct strongly interacting particles is not meaningful.[12]

The new results we have obtained by including the strong interactions via I.PA.M. are: (1) the neutron has an effective mass of 0.9 and the proton has an effective mass of 0.6 for densities not too different[13] from nuclear

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density; and (ii) the threshold densities for producing various hadrons are differentially shifted from the threshold values predicted by the noninteracting gas model.

In order to illustrate the importance of effective masses we note that the number density of electrons (or protons) in the n-p-e phase is proportional to $(m_n^*)^{-3}$ (where m_n^* = the effective mass of the neutron), the threshold density for producing muons is proportional to $(m_n^*)^{+3/2}$, and the Fermi energy of the protons is proportional to $(m_n^*)^{-2}$ $(m_p^*)^{-1}$. The reason for the shifts in the threshold densities of hadrons is most clearly understood by considering a specific example in which the self-consistent, single-particle potential experienced by each strongly interacting particle is approximated by a quadratic expression. Thus sigmas are produced at densities such that $(e^- + n \rightarrow \Sigma^- + \nu_e)$:

$$E_F(e) + \frac{P_F^2(n)}{2m_n^*} \geq (m_\Sigma - m_n) + (B_0(\Sigma) - B_F(n)) , \qquad (2)$$

where the terms on the left-hand side of Eq. (2) are the electron and neutron Fermi energies, respectively, $B_0(\Sigma)$ is the binding energy of a zero energy Σ^-, and $B_F(n)$ is the average potential energy of a neutron at the top of its Fermi sea. Equation (2) reduces to the usual result[3,8,14] if m_n^* is replaced by m_n and the term $(B_0(\Sigma) - B_F(n))$ is ignored.

Pions are produced at densities such that $(e^- + n \rightarrow \pi^- + \nu_e + n')$:

$$E_F(e) \geq m_\pi + B_0(\pi^-) \qquad (3)$$

In the noninteracting gas model, Σ^-'s are produced at much lower densities than π^-'s.[3,8,14] One can show with Eqs. (2) and (3) that pions are actually produced at lower densities than sigmas if

$$B_0(\pi^-) \lesssim -20 \text{ MeV} + \left(\frac{B_0(\Sigma^-) - B_0(p)}{2}\right), \tag{4}$$

for stellar densities $\lesssim$ three times nuclear densities. On the basis of hyperfragment data, one can make the conservative statement that $(B_0(\Sigma^-) - B_0(p))/2 \gtrsim -5$ MeV. Hence criterion (4) becomes $B_0(\pi^-) \lesssim -25$ MeV.

Miyazawa[15] has recently calculated, with the aid of general field theoretic arguments, the difference between the pion self-energy in nuclear matter and in vacuum; his result is expressed in terms of the known cross sections for π-N scattering in vacuum. This calculation, when adapted[9] to the case of a neutron star, indicates that $B_0(\pi^-)$ is much less than -25 MeV for densities $\gtrsim$ nuclear densities. Thus pions are produced before sigmas and the numbers of particles of various kinds present in neutron stars are probably very different from the numbers previously obtained[3,8,14] by neglecting the strong interactions. The practical importance of this result is that the presence of a significant number of pions in a neutron star changes[16] the predicted cooling rates of a hot neutron star by a tremendous factor ($\sim 10^{+8}$), as we shall see in the following paragraphs.

In order to compute cooling times, one must consider the various excited states of a neutron star. One can imagine that these excited states are populated (according to the usual Boltzmann factor) by placing the system in contact with a thermal bath at a finite temperature T. The star then has a definite baryon number and total electric charge but does not have a definite energy. The rate of energy loss (cooling) by neutrino emission is given by an expression of the form:

$$L_\nu = \text{const.} \sum_\nu \sum_{\beta<\alpha} \sum_\alpha |(S_\beta;\nu\,|H_\nu|\,S_\alpha)|^2\, E_\nu \delta(E_\alpha - E_\beta - E_\nu) \exp(-E_\alpha/kT), \tag{5}$$

where S_α, S_β are states of the entire star, H_w is the weak interaction Hamiltonian, E_ν is the energy of the neutrino, ν, that is radiated, and the summation over β is limited to states for which $E_\beta < E_\alpha$.

In practice, cooling times must be computed by assuming a model; we adopt I.PA.M. We also approximate the thermal average (Eq. (5)) over the states of the star by assigning a Fermi-Dirac or Bose-Einstein distribution to each kind of particle in the star. The most important cooling reactions are:

$$n + n \to n' + p + e^- + \bar{\nu}_e \; ; \qquad (6a)$$

$$n + \pi^- \to n' + \mu^- + \bar{\nu}_\mu \; ; \qquad (6b)$$

and their inverses, $p + e^- + n' \to n + n + \nu_e$ and $n' + \mu^- \to n + \pi^- + \nu_\mu$. Unlike photons, neutrinos produced in the interior of a neutron star escape from the surface with a negligible probability[17] of having been absorbed or scattered. Reaction rates for processes (6) and their inverses (which have equal rates within our approximations) have been estimated[9] by distorted-wave Born approximations using empirically determined scattering potentials and weak interaction matrix elements. The exclusion principle for Fermions was included in the phase-space with a Fermi-Dirac distribution function. We find for the rate of energy loss from (6a) and its inverse:

$$L_\nu(\text{ergs/sec}) \cong 10^{+38} \left(\frac{M}{M_\odot}\right)\left(\frac{T_c}{10^{+9}\ {}^\circ K}\right)^8 \left(\frac{\rho_{\text{nucl}}}{\rho}\right) \text{erg - sec}^{-1} \qquad (7a)$$

and from (6b) and its inverse:

$$L_\nu(\text{ergs/sec}) \cong 10^{+46} \left(\frac{n_\pi}{n_n}\right)\left(\frac{M}{M_\odot}\right)\left(\frac{T_c}{10^{+9}\ {}^\circ K}\right)^6 \left(\frac{\rho_{\text{nucl}}}{\rho}\right)^2 \text{erg - sec}^{-1}\,, \qquad (7b)$$

where M, $M_\odot$, T_c, ρ, ρ_{nucl}, n_π, and n_n are, respectively, the mass of the neutron star, the mass of the sun, the central temperature of the star, the central density of the star, $3.7 \times 10^{+14}$ gm cm^{-3}, the number density of pions, and the number density of neutrons. Our rate, (7a), for cooling by reactions (6a) is two orders of magnitude faster, in the important temperature-density range, than the rate estimated by Chiu and Salpeter.[2,18] One can show, by using the models of neutron stars given in refs. (1) and (3), that the neutrino luminosity from reactions (6a) exceeds the photon luminosity for effective (i.e., surface) temperatures $\gtrsim$ two million degrees. The energy loss, Eq. (7b), from neutrino emission by pions is much greater than from all previously known cooling processes; this fast cooling rate is expected to apply for a density $\gtrsim \rho_{nucl}$, which is close to the minimum density required for the stability of a neutron star on any current model. The basic reason that the energy loss from reactions (6b) is so much faster than from, for example, reactions (6a) is that pions are bosons. Each fermion that participates in a cooling reaction introduces a factor in the cooling rate of $kT/E_F \sim 10^{-3}$ to 10^{-4}, where E_F is the Fermi energy of the particle; this factor occurs because only the small fraction ($\sim kT/E_F$) of the fermions that are on the tail of the Fermi-Dirac distribution can make transitions allowed by the exclusion principle. No such restriction exists for bosons. Note that reaction (6a) involves two more fermions than reaction (6b)

The cooling rate, (7b) is so great that it seems very unlikely that neutron stars will be observable with present techniques[5-7] if, as indicated by our adaptation of the work of Miyazawa,[15] $B_0(\pi^-) \lesssim -25$ MeV. For example, one can show with the models of ref. (3) that the x-ray source in Scorpius, which Friedman[7] indicates might have a surface temperature of the order of 2 or 3 million degrees, would decrease in photon luminosity by a factor of ten in a period of less than or of the order of a week if Eq. (6b) applies. This result strongly suggests that the source in Scorpius is not a neutron star. Our fast cooling rates are also inconsistent with the hypothesis that a hot neutron star exists in the Crab Nebula which is a remnant of a supernova explosion that occurred in 1054.

Models of the type (I.PA.M.) used in our calculations, which rely heavily on the concept of individual particles, will fail at densities $\gtrsim$ 10 times nuclear densities (cf. Eq. (1)). In particular, the distinction between fermions and bosons probably disappears at such high densities that hadrons are continually within a hard core distance of each other.[12] Thus the tables that some authors have given which describe the state of a neutron star in terms of the number of, e.g., Ξ^-'s present at $\rho > 25\ \rho_{nucl}$ are unjustified. At $\rho_{stellar} > 10\ \rho_{nucl}$, it presumably makes sense to specify a definite baryon number and charge, but one can at best hope to calculate, for example, an expectation value for the strangeness per unit volume. The problem of how to describe the state of matter at very high densities correctly and in a manner suitable for calculation is fascinating but unsolved. In any event, the hadronic constituents of matter (if this phrase continues to have an approximate meaning) at very high densities will be vastly different from their free-particle analogues.

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12. Imagine, for example, a collection of alpha particles at a density for which $d > R_\alpha$, where R_α is the "radius" of an alpha particle. If the density of alpha particles is now increased so that $d \lesssim R_\alpha$, the alpha particles will come apart into their constituents, primarily neutrons and protons.

13. There is a slight variation of the neutron and proton effective masses with density. This variation can be estimated (ref. 9) from the work of K. A. Brueckner, Phys. Rev. 97, 1353 (1955). The effective masses of the neutron and proton are different in a neutron star because the stellar matter contains many more neutrons than protons.

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16. The large number of pions present at $\rho \gtrsim \rho_{nucl}$ form a degenerate Bose gas; their presence affects (ref. 9) the theoretical equation of state (lowers the pressure) and the sequence in which various particles are produced.

17. The neutrino opacity of a neutron star can be determined from formulae given by J. N. Bahcall, Phys. Rev. 136, B1164 (1964) and J. N. Bahcall and S. C. Frautschi, Phys. Rev. 136, B1547 (1964). The largest contribution to the opacity (for neutrinos from reaction (6a)) arises from neutrino-electron scattering. When applying formula (57) of the paper by Bahcall, note that $\mu_e > 10$ for a neutron star.

18. A. Finzi, Phys. Rev. (to be published) has calculated the rate of reaction (6a) for a specific density, $\rho = 1.5\ \rho_{nucl}$. He treats the neutrons and protons as free particles (neglecting nuclear-matter effects) and estimates, in second order perturbation theory, the energy loss due to pairs of successive virtual transitions. We have used, on the other hand, first order perturbation theory with empirically determined matrix elements and potentials and have included, in an approximate way, nuclear matter effects. However, our result for $\rho = 1.5\ \rho_{nucl}$ is in surprisingly good agreement with his value.